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The Infrared Polarization of Sea Radiance

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EXECUTIVE SUMMARY

OBJECTIVE

Predict the polarization dependence of infrared sea radiance (brightness).

APPROACH

The Fresnel coefficients are applied to the reflection of unpolarized monochromatic radiation by a sea water capillary wave facet. The result for a single tilted facet is integrated over all possible tilts and wave numbers to predict the polarization dependence of radiance received from the wind-ruffled ocean surface.

RESULTS

Equations valid in the infrared are derived for horizontal, vertical, and unpolarized sea radiance. They show the following regularities: (1) horizontal and vertical sea reflectivity sum to the unpolarized sea reflectivity, (2) horizontal and vertical sea emissivity sum to the unpolarized sea emissivity, and (3) horizontal and vertical sea radiance sum to the unpolarized sea radiance.

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INTRODUCTION

When a linearly polarized plane wave of electromagnetic radiation is incident on the boundary between two homogeneous materials with different optical properties, it is well known that the reflected plane wave is described by the Fresnel equations (Born & Wolf, 1970, and Stratton, 1941). The Fresnel equations arise from requiring a match between the components of Maxwell's equations on either side of the boundary. The Fresnel equations show that the amplitudes and phases of the reflected waves are different for waves incident with electric vectors parallel ("p") and perpendicular ("s") to the plane of incidence[1], and, furthermore, that these two components are mathematically separable and therefore independent of one another. When the two components are recombined, differences that have arisen between them during reflection lead to elliptically polarized light. Electromagnetic theory thus shows that a surface introduces a change in the state of polarization of electromagnetic radiation.

THE PROBLEM

The purpose of this report is to explain how the equations for Fresnel reflection can be used to predict the polarization of sea radiance. The experiment we wish to consider is as follows: (1) Observe the radiance (brightness) of the sea with an instrument that responds in the infrared region of the electromagnetic spectrum. (2) Provide for the insertion and removal of a polarizer in front of the radiometer. If the polarizer is perfect, it will completely transmit radiation whose electric vector is parallel to its axis of polarization, and the polarizer will completely block radiation whose electric vector is perpendicular to its axis of polarization. (3) Point the instrument toward the sea and make three measurements: N_h, the radiance with the polarizer axis horizontal; N_v, the radiance with the polarizer axis vertical; and N, the radiance with the polarizer removed.

The problem is to predict the values of N_h, N_v, and N. The solution to this problem has already been given (Gregoris et al., 1992), and comparisons have been made with data (Cooper et al., 1992). In this report, we supply an analysis to clarify the physics involved.

[1] The plane of incidence contains the incident ray, the boundary normal, and the reflected ray.

THE COORDINATE SYSTEM

We will analyze the problem in successive stages using the notation of Stratton (1941) as much as possible. First, we consider the reflection of monochromatic radiation from a flat sea water surface tilted with respect to the horizon. This case represents the situation that is assumed to occur when a single capillary wave facet whose radius of curvature is much greater than the wavelength of the radiation is exposed to the sky or sun (Walker, 1994 and Zeisse, 1995a & 1995b). Figure 1 shows the geometry of reflection. Since the distribution of capillary wave slopes depends somewhat on the wind direction it will prove to be convenient for later analysis if the coordinate system has a definite orientation with respect to the azimuth of the wind. A coordinate system has been chosen whose origin is the point of reflection with the *X*-axis pointing up wind, the *Y*-axis pointing cross wind, and the *Z*-axis pointing toward the zenith, such that a right-handed system is formed. The *X*–*Y* plane is horizontal at the point of reflection. The tilted facet passes through the origin.

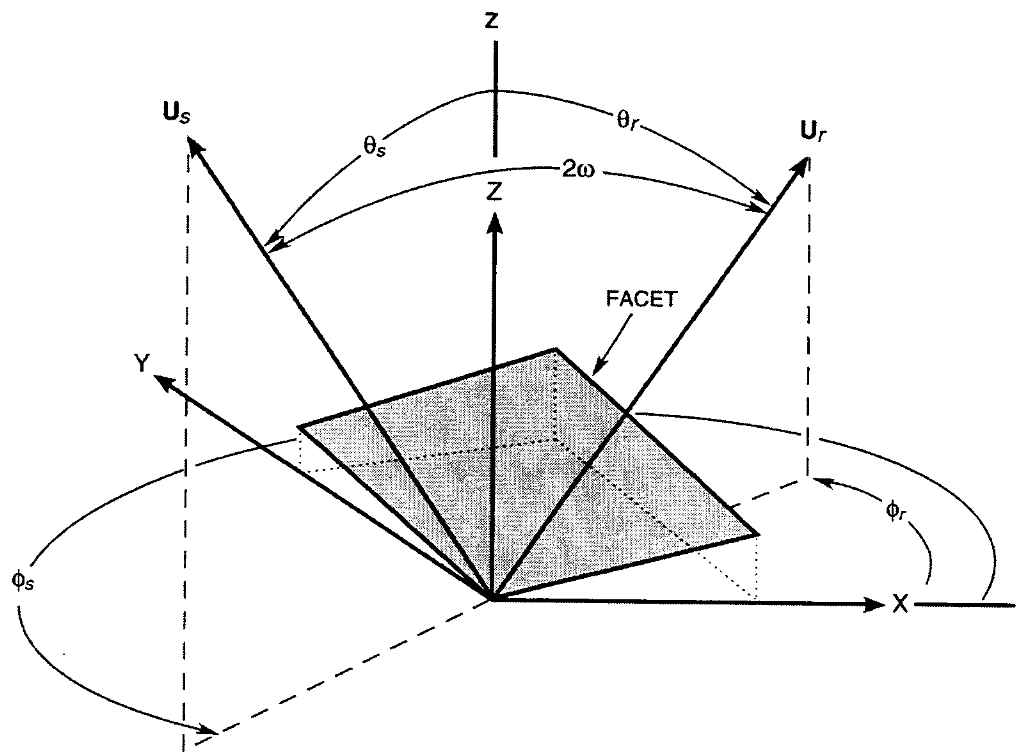

Figure 1. The coordinate system. The tilted facet passes through the origin at the point of reflection. The *X*–*Y* plane is horizontal. Unit vector $\mathbf{U}_s$ points to the source. Unit vector $\mathbf{U}_r$, coincident with the optical axis of the polarizer, points to the receiver.

Define a unit vector $\mathbf{U} \equiv (\theta, \phi)$ with polar coordinates θ as the zenith angle, and ϕ as the azimuth. If we denote the Cartesian coordinates of $\mathbf{U}$ by (a, b, c), then we have

$$\begin{aligned} a &= \sin\theta\,\cos\phi \\ b &= \sin\theta\,\sin\phi \\ c &= \cos\theta \end{aligned} \tag{1}$$

Two such unit vectors are shown in figure 1: $\mathbf{U}_s$, pointing from the origin to the source, and $\mathbf{U}_r$, pointing from the origin to the receiver. A third unit vector, $\mathbf{U}_n$, is normal to the facet at the point of reflection but was removed from the figure for clarity[2]. When there is a specular reflection at the facet, then

$$\mathbf{U}_s + \mathbf{U}_r = 2\cos\omega\,\mathbf{U}_n \tag{2}$$

where ω is the angle of incidence and the angle of reflection.

The facet slope in the upwind direction, ζ_x, is given by the slope of the line formed at the intersection of the facet with the X–Z plane. The facet slope in the cross-wind direction, ζ_y, is given by the slope of the line formed at the intersection of the facet with the Y–Z plane. In Cartesian coordinates these slopes are

$$\begin{aligned} \zeta_x &= -a_n/c_n \\ \zeta_y &= -b_n/c_n \end{aligned} \tag{3}$$

[2] The zenith angle of $\mathbf{U}_n$ is the same as the tilt of the facet. The tilt is the angle of the steepest ascent within the facet.

INCIDENT AMPLITUDES

We will imagine initially that the incident electromagnetic wave arises from a single radiating atom in the source so that the incident wave front is linearly polarized and perfectly coherent. The incident **E** vector, $\mathbf{E}_o$, will have a unique direction in this instance, and we denote by α the angle of arrival, the angle that $\mathbf{E}_o$ makes with the plane of incidence (see figure 2). The incident vector can be resolved into two components: E_{os} perpendicular to the plane of incidence and E_{op} parallel to the plane of incidence. These incident amplitudes are shown in figure 2 where it is apparent that

$$\begin{aligned} E_{os} &= E_o \sin\alpha \\ E_{op} &= E_o \cos\alpha \end{aligned} \tag{4}$$

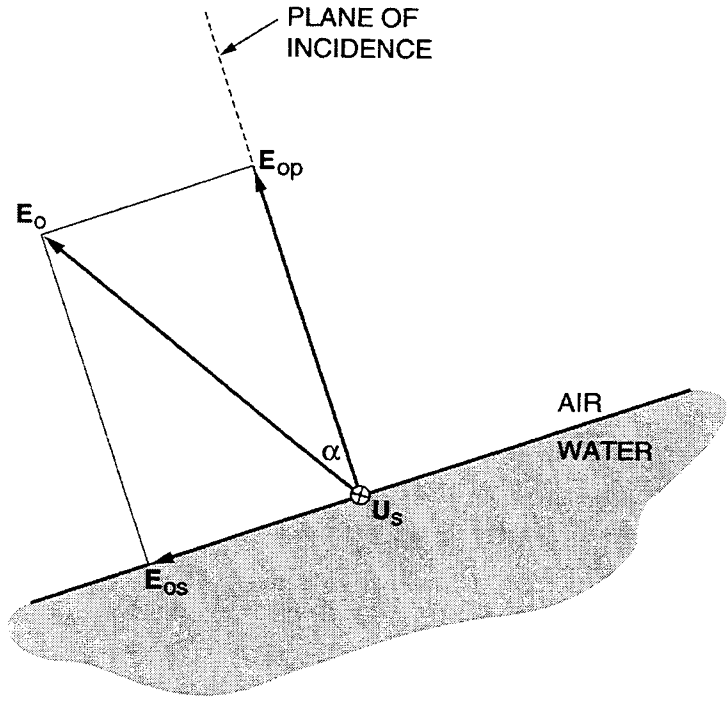

Figure 2. Incident coherent amplitudes. The view is along vector $\mathbf{U}_S$ toward the source.

REFLECTED AMPLITUDES

Next, the Fresnel equations are applied to each incident component to give the reflected amplitudes and phases (figure 3):

$$
\begin{aligned}
E_{2s} &= \rho_s \exp(-i\delta_s) E_{os} \\
E_{2p} &= \rho_p \exp(-i\delta_p) E_{op}
\end{aligned}
\tag{5}
$$

We assume that sea water is a conducting medium for which $\delta_s \neq \delta_p$. Equations for ρ_s, δ_s, ρ_p, and δ_p, are given in terms of the optical constants of air (medium 2) and water (medium 1) by equations derived in Stratton (1941). These equations are reproduced as (A–9) through (A–11) in Appendix A.

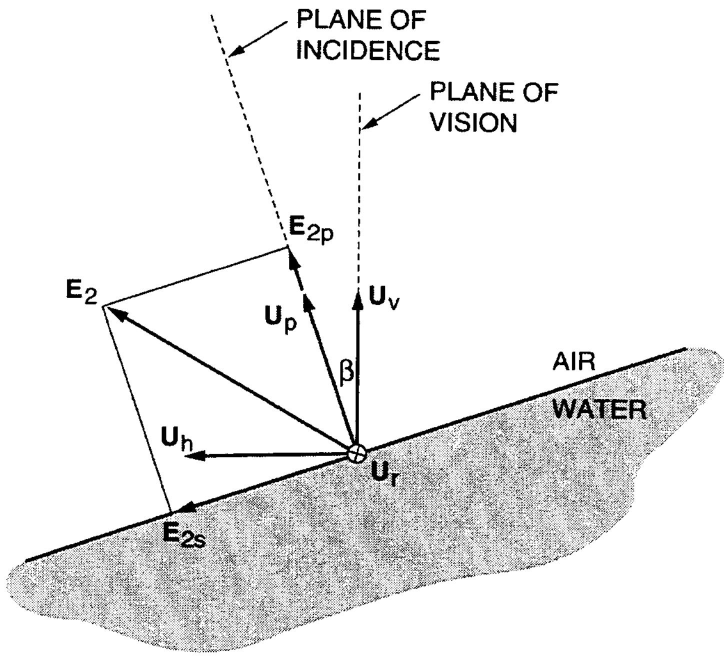

Figure 3. Reflected coherent amplitudes. The view is against vector $\mathbf{U}_r$ toward the facet. All vectors shown here are normal to $\mathbf{U}_r$ in the plane of the paper.

The polarizer has an optical axis along which the radiation passes and, normal to this optical axis, a polarization axis that is the axis of "easy passage" for the **E** vector. Let the optical axis of the polarizer coincide with the direction of $\mathbf{U}_r$ in figure 1 and set the polarization axis either normal to the plane of vision[3] (along the unit vector $\mathbf{U}_h$ in figure 3) or parallel to the plane of vision (along the unit vector $\mathbf{U}_v$ in figure 3). In each position only those components of $\mathbf{E}_{2s}$ and $\mathbf{E}_{2p}$ parallel to the polarization axis are transmitted along the optical axis. The horizontal, vertical, and unpolarized amplitudes will be given respectively by

[3] The plane of vision (Gregoris, 1992) contains the vectors $\mathbf{U}_r$ and $\mathbf{k}$, where $\mathbf{k}$ is a unit vector along the Z axis.

$$
\begin{aligned}
E_{2h} &= +E_{2s}\cos\beta + E_{2p}\sin\beta \\
E_{2v} &= -E_{2s}\sin\beta + E_{2p}\cos\beta \\
\mathbf{E}_2 &= +\mathbf{E}_{2s} + \mathbf{E}_{2p}
\end{aligned}
\tag{6}
$$

In Appendix C, it is shown that β, the angle between the plane of incidence and the plane of vision, is given by

$$
\sin\beta = \frac{\sin(\phi_r - \phi_n)\sin\theta_n}{\sin\omega} \tag{7}
$$

where ϕ_r is the azimuth of the receiver ray (figure 1), ϕ_n is the azimuth of the facet normal, θ_n is the zenith angle of the facet normal, and ω is the angle of incidence.

COHERENT REFLECTED INTENSITIES

Now that we have expressions in (6) for the reflected amplitudes, the intensities are found by computing the squared modulus $E^{*} \cdot E$ for each case. Applying this to the amplitudes given in (6) and using (5) and (4) we find the corresponding intensities

$$
\begin{aligned}
E_{2h}^{*} \cdot E_{2h} &= \left[R_s(\sin\alpha\cos\beta)^2 + (1/2)\rho_s\rho_p \sin 2\alpha \sin 2\beta \cos\delta + R_p(\cos\alpha\sin\beta)^2\right] E_o^{*} \cdot E_o \\
E_{2v}^{*} \cdot E_{2v} &= \left[R_s(\sin\alpha\sin\beta)^2 - (1/2)\rho_s\rho_p \sin 2\alpha \sin 2\beta \cos\delta + R_p(\cos\alpha\cos\beta)^2\right] E_o^{*} \cdot E_o \qquad (8) \\
E_2^{*} \cdot E_2 &= \left[R_s(\sin\alpha)^2 + \rho_s\rho_p \sin 2\alpha \cos\delta + R_p(\cos\alpha)^2\right] E_o^{*} \cdot E_o
\end{aligned}
$$

In equations (8), $R_s \equiv \rho_s{}^2$, $R_p \equiv \rho_p{}^2$, and $\delta \equiv \delta_p - \delta_s$ are given by equations (A–9) through (A–12). R_s and R_p are the Fresnel reflection coefficients. They are shown for sea water in figures 4 and 5 for wave numbers of 1000 cm^{-1} (10 μm) and 2500 cm^{-1} (4 μm), respectively. Appendix B contains the FORTRAN program used for the calculation shown in figures 4 and 5. The program, taken from the equations of Stratton and employing his notation for the most part, is a simplified version of the complete treatment (summarized in Appendix A) that nevertheless contains all the essential ingredients for the present situation.

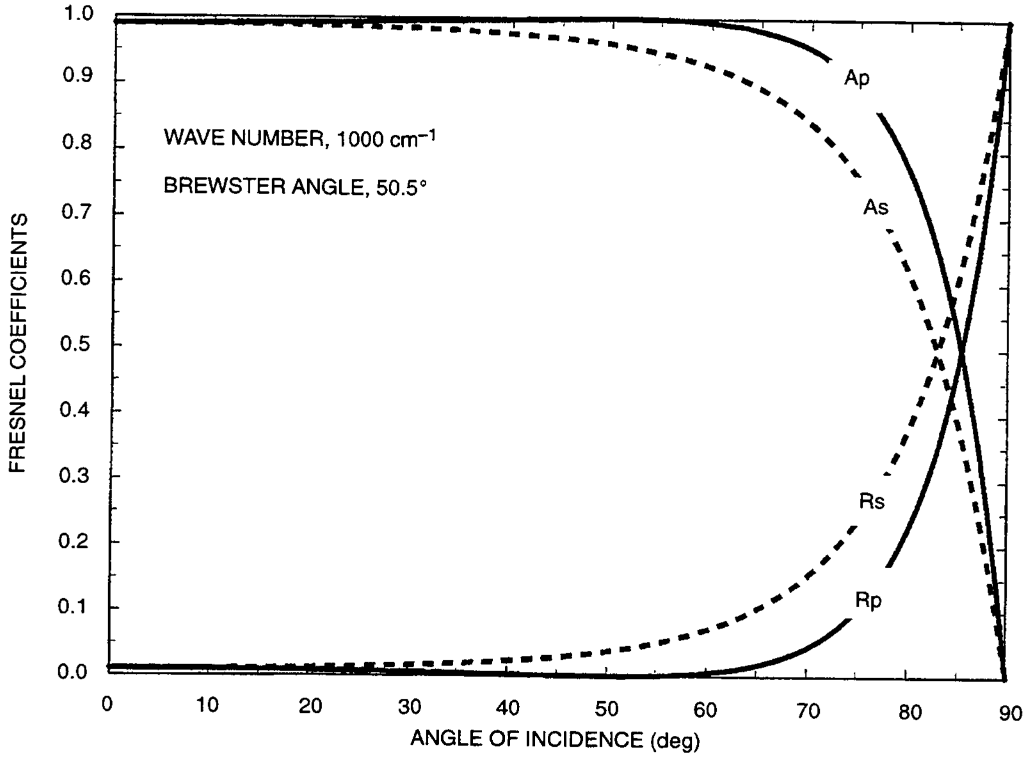

Figure 4. Sea water Fresnel coefficients at a wave number of 1000 cm^{-1} (a wavelength of 10 μm).

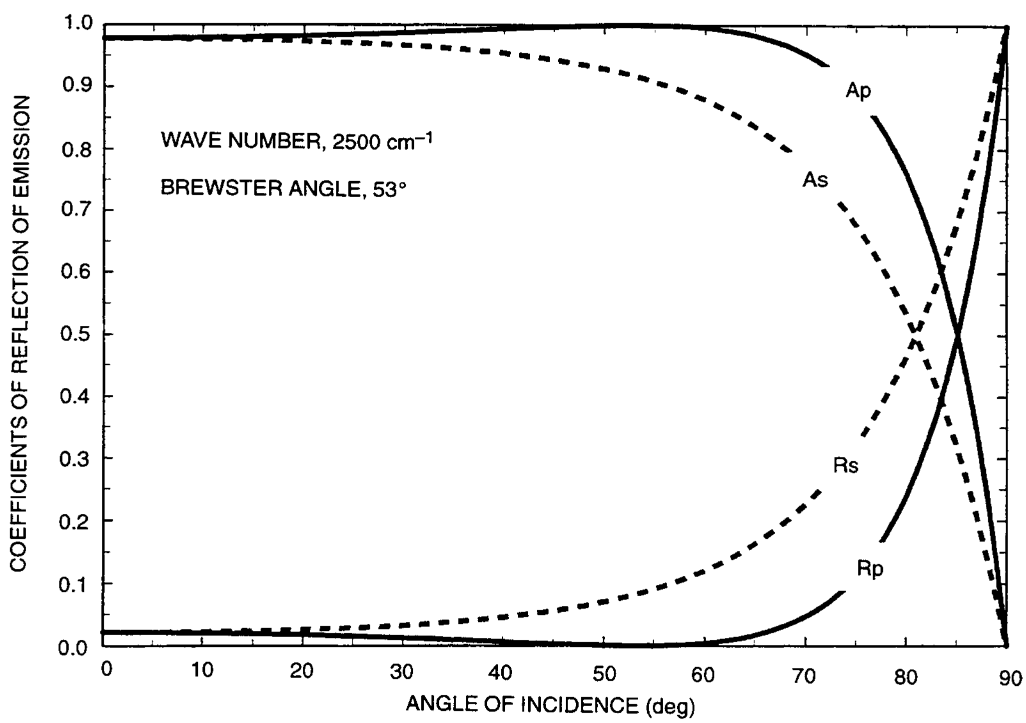

Figure 5. Sea water Fresnel coefficients at a wave number of 2500 cm^{-1} (a wavelength of 4 μm).

NATURAL REFLECTED INTENSITIES

Up to this point, we have been considering the reflection of light emitted by a single atom. For this case the cross terms in (8), those proportional to the product of ϱ_s and ϱ_p, represent the effects of interference between the two components of the reflected wave. These cross terms vanish when the incident light is natural for the following reason. A natural source will contain many atoms, each radiating independently of another with an incident amplitude that will vary about some average amplitude and an angle of arrival that will vary at random. Furthermore, the amplitude E_o of a natural source will not depend on the angle of arrival, α, that is, a natural source will be unpolarized. If we wish to apply equations (8) to a natural source, then we must average (8) over all angles of arrival. Since the averages of $\sin^2\alpha$ and $\cos^2\alpha$ are ½ and the average of $\sin 2\alpha$ vanishes (destroying the interference effects), equations (8) for the reflection of natural light become

$$
\begin{aligned}
R_h &\equiv \frac{N_h^{sky}}{N_o} = \frac{R_s}{2}(\cos\beta)^2 + \frac{R_p}{2}(\sin\beta)^2 \\
R_v &\equiv \frac{N_v^{sky}}{N_o} = \frac{R_s}{2}(\sin\beta)^2 + \frac{R_p}{2}(\cos\beta)^2 \\
R &\equiv \frac{N^{sky}}{N_o} = \frac{R_s}{2} + \frac{R_p}{2}
\end{aligned}
\tag{9}
$$

where the reflected radiance N^{sky} is assumed to be proportional to $E_2{}^{*}\cdot E_2$ for each polarization condition and the incident sky radiance N_o is assumed to be proportional to the incident intensity $E_o{}^{*}\cdot E_o$.

Equations (9) can be expected to hold for wave numbers up to and including those in the infrared. However, the assumption made in the previous paragraph that the radiance falling on the sea is unpolarized will fail as we approach the visible because of the celebrated effect of Rayleigh scattering (McCartney, 1976), dipole scattering from irregularly spaced molecules in the atmosphere that, in addition to introducing a blue color to the sky, introduces polarization to the sky radiance. In Appendix D we estimate that the total polarization of the sky reaches 1 percent at a wave number of 3500 cm^{-1} (a wavelength of 2.9 μm).

It is evident from (9) that

$$R_h + R_v = R \tag{10}$$

independent of the value of β, that is, independent of the tilt of the capillary wave facet. This implies that, even for measurements made on the real sea containing capillary wave facets of many different slopes, the sum of the measurements with the polarizer inserted equals the measurement with the polarizer removed (assuming a perfect polarizer).

We pause here to emphasize a fact obscured by our notation, namely that the Fresnel reflection coefficients, R_s and R_p, have the *component* intensity in their denominator whereas the measured reflection coefficients, R_h, R_v, and R, have the *total* intensity in their denominator.

NATURAL EMITTED INTENSITIES

The electromagnetic theory alluded to in our introductory paragraph shows that under the conditions we are considering in this problem, plane waves incident on a plane boundary between two homogeneous media, the equations for reflection are separable into two independent "*s*" and "*p*" components. In each of these components, the reflection coefficients represent the ratio of reflected energy to incident energy; indeed it can be confirmed by inspection of the complicated equations for R_s and R_p (Stratton, 1941) that each of these quantities is always less than or equal to one. The remaining energy, the energy not reflected at the interface, can be found immediately across the interface inside the other medium (water) where it has been transmitted by the boundary and where its value is represented by Fresnel transmission coefficients that we will not discuss. The transmitted energy is attenuated by absorption as it propagates away from the interface.

This propagation and attenuation might continue until a second interface was encountered where a new reflection and transmission would occur very similar to the one we have been considering. But now we are involved in a slightly different problem, the problem of reflection by a thin parallel plate sample of water. Let us return to our original case for which the water is thick; none of the energy can escape by transmission; and all the energy is absorbed within the water itself. Absorption converts the optical energy into heat; the water is heated; the black body emission of the water is increased; and we are implicitly involved in another new development, a non-equilibrium heating of the water by the light. This process would continue until thermal equilibrium was established at some higher temperature where optical absorption was balanced by thermal emission. This balance is expressed by Kirchoff's Law (Born & Wolf, 1970, page 622), which says that for each wave number (and, we assume here, for each component "*s*" and "*p*") the energy emitted by the water, divided by the black body radiance of the water N_{bb}, equals the fraction A that the water absorbs of the radiant energy which falls upon it:

$$\begin{aligned} \frac{N_s^{sea}}{N_{bb}} &= A_s = 1 - R_s \\ \frac{N_p^{sea}}{N_{bb}} &= A_p = 1 - R_p \end{aligned} \tag{11}$$

The Fresnel emission coefficients A_s and A_p are shown in figures 4 and 5.

Since the polarizer acts upon these emitted intensities in the same way that it acts upon reflected intensities, the horizontal, vertical, and unpolarized emitted intensities, A_h, A_v, and A, are given by (9) with R_s replaced by A_s, and R_p replaced by A_p:

$$\begin{aligned} A_h &\equiv \frac{N_h^{sea}}{N_{bb}} = \frac{A_s}{2}(\cos\beta)^2 + \frac{A_p}{2}(\sin\beta)^2 = \frac{1}{2} - R_h \\ A_v &\equiv \frac{N_v^{sea}}{N_{bb}} = \frac{A_s}{2}(\sin\beta)^2 + \frac{A_p}{2}(\cos\beta)^2 = \frac{1}{2} - R_v \\ A &\equiv \frac{N^{sea}}{N_{bb}} = \frac{A_s}{2} + \frac{A_p}{2} = 1 - R \end{aligned} \tag{12}$$

In (12), $N^{sea}{}_h$, $N^{sea}{}_v$, and N^{sea} represent, respectively, the emitted radiance detected with the polarizer normal to the plane of vision ("horizontal"), within the plane of vision ("vertical"), and removed. Equations (12) show that the horizontal and vertical emissions also sum to the unpolarized emission in the real sea (for a perfect polarizer).

We wish to comment on a change of emphasis brought about within (12). In those equations, A and R are energy fractions that add to constant values, expressing the conservation of energy (incident energy is absorbed or reflected). However, we have used Kirchoff's Law to replace absorbed energy with emitted energy, giving A and R different denominators (black body energy and incident energy, respectively). After this switch, we are only interested in the numerators by themselves; hence, there is nothing to be gained in the observation that, for example, $A_v + R_v = \frac{1}{2}$, even though this happens to be true.

INTEGRATION OVER SLOPES

We now take the expressions for spectral sea radiance we have derived for a single-wave slope, apply a weight for that slope, and integrate over all slopes. Applying this to (9) and (12) gives:

$$
\begin{aligned}
N_h^{sky} &= \langle N_o R_h \rangle_r \\
N_v^{sky} &= \langle N_o R_v \rangle_r \\
N^{sky} &= \langle N_o R \rangle_r
\end{aligned}
\tag{13}
$$

for the three radiance values arising from sky or sun reflections and

$$
\begin{aligned}
N_h^{sea} &= N_{bb} \left\langle \frac{1}{2} - R_h \right\rangle_r \\
N_v^{sea} &= N_{bb} \left\langle \frac{1}{2} - R_v \right\rangle_r \\
N^{sea} &= N_{bb} \langle 1 - R \rangle_r
\end{aligned}
\tag{14}
$$

for the three radiance values arising from thermal sea emission.

In each equation of (13) and (14), the angular brackets represent the following integral over slope space (Zeisse, 1995b):

$$
\langle f \rangle_r \equiv \frac{\displaystyle\iint\limits_{\substack{\omega \le \pi/2 \\ U_r = const}} f \, \frac{\cos\omega}{\cos\theta_n} p(\zeta_x, \zeta_y, W) \, d\zeta_x \, d\zeta_y}{\displaystyle\iint\limits_{\substack{\omega \le \pi/2 \\ U_r = const}} \frac{\cos\omega}{\cos\theta_n} p(\zeta_x, \zeta_y, W) \, d\zeta_x \, d\zeta_y}
\tag{15}
$$

where, following Cox and Munk (1954), $p(\zeta_x, \zeta_y, W)$ stands for the probability that a wave facet will occur with a slope within $\pm d\zeta_x/2$ of ζ_x and $\pm d\zeta_y/2$ of ζ_y when the wind speed is W. Cox and Munk (1954 & 1956) obtained an expression for p, the occurrence probability density, in which the lowest order term is

$$
\begin{aligned}
p(\zeta_x, \zeta_x, W) &\approx \frac{1}{2\pi\sigma_u\sigma_c} \exp\left\{ -\frac{1}{2}\left[\frac{\zeta_x^2}{\sigma_u^2} + \frac{\zeta_y^2}{\sigma_c^2} \right) \right] \right\} \\
\sigma_u^2 &= 0.000 + 3.16 \cdot 10^{-3} W \\
\sigma_c^2 &= 0.003 + 1.92 \cdot 10^{-3} W
\end{aligned}
\tag{16}
$$

In equation (16), $\sigma_u{}^2$ and $\sigma_c{}^2$ are the variances in ζ_x and ζ_y, respectively, and W is the wind speed in m s^{-1}.

The factor cos ω/cos θ_n in (15) projects the horizontal facet area toward the receiver, weighting each facet by the amount of that facet "seen" at the receiver. This weight is the central assumption of a sea-radiance model, contained in equations (13) through (16), which has been more fully described elsewhere (Zeisse, 1995c). The subscript "r" on the angular bracket serves as a reminder that the receiver direction $\mathbf{U}_r$ is held fixed during the integration over sea slopes ζ_x and ζ_y.

In each equation of (13), $N_o(\theta_s, \phi_s, \nu)$ represents the spectral radiance from the sky or sun incident on the facet from source coordinates (θ_s, ϕ_s) that are related to the (fixed) receiver coordinates and the ocean slopes by the law of specular reflection. When the radiance is from the sky, the integration is over all slopes in the sea capable of reflecting the sky[4] into the receiver. When the radiance is from the sun, the integration is only over the tolerance ellipse (those slopes capable of reflecting part of the solar disk toward the receiver). In (14), $N_{bb} = N_{bb}(T_{sea}, \nu)$, the integration is over all slopes in the sea, and no specular reflection is considered to occur.

[4] Some slopes might be tilted so that arriving radiance would have to originate below the horizon ($\theta_s > 90°$). These are excluded. As a matter of fact, shadowing and multiple reflections make the integration unrealistic when radiance arrives from within about 10° of the horizon ($\theta_s > 80°$)

THE SOLUTION

The previous discussion has been concentrated solely on reflection by capillary wave facets within the footprint of a single pixel from an image of the ocean surface. Incident values have been those arriving at the footprint, and reflected values have been those immediately leaving the footprint. In most practical situations, however, the receiver is located a significant distance away from the footprint, typically several km. Along the path to the receiver, the reflected and emitted radiance are attenuated, and path radiance is accumulated.

The path radiance is unpolarized natural light. What will its intensity be when observed through the polarizer? The argument follows along the lines of those employed earlier. If we consider a single atom in the radiating atmosphere, then we may describe its radiation as a linearly polarized coherent plane wave propagating along the optical path to the receiver. The incident vector amplitude, $\mathbf{E}_o$, will have a unique direction in this instance, and we let α denote the angle that the amplitude $\mathbf{E}_o$ makes with the axis of the polarizer, whatever its orientation. The polarizer will transmit the amplitude $E_o \cos \alpha$ corresponding to an intensity of $E_o^* \cdot E_o \cos^2 \alpha$. Averaging over all values of α to represent all atoms radiating in this natural light source, we conclude that a polarizer, if perfect, would transmit half of the unpolarized value regardless of its orientation.

Gathering all these results and combining reflected and emitted intensities in each direction, we have this solution for the sea radiance observed with the polarizer horizontal, vertical, and removed:

$$
\begin{aligned}
N_h &= \left\{ \left\langle N_o \frac{R_s \cos^2 \beta + R_p \sin^2 \beta}{2} \right\rangle_r + N_{bb} \left\langle \frac{1 - R_s \cos^2 \beta - R_p \sin^2 \beta}{2} \right\rangle_r \right\} \tau f + \frac{N^{path} f}{2} \\
N_v &= \left\{ \left\langle N_o \frac{R_s \sin^2 \beta + R_p \cos^2 \beta}{2} \right\rangle_r + N_{bb} \left\langle \frac{1 - R_s \sin^2 \beta - R_p \cos^2 \beta}{2} \right\rangle_r \right\} \tau f + \frac{N^{path} f}{2} \qquad (17) \\
N = N_h + N_v &= \left\{ \left\langle N_o \frac{R_s + R_p}{2} \right\rangle_r + N_{bb} \left\langle 1 - \frac{R_s + R_p}{2} \right\rangle_r \right\} \tau \; f + N^{path} \; f
\end{aligned}
$$

Here the values of R_s and R_p should be taken from equations (A-9), (A-10), and (A-11), and equation (7) should be used for β. The spectral transmission along the path from the footprint to the receiver is represented in (17) by the symbol τ. The function f has been included in (17) to represent the relative spectral responsivity of the instrument. All quantities except β in (17) depend on the wave number ν. Broad-band results are obtained by integrating (17) over all wave numbers.

SEA RADIANCE RELATIONSHIPS

It is convenient to separate the radiance N_o incident on each facet into the radiance arriving from the sun and the radiance arriving from the sky. Then, the first term in each equation of (17) can be similarly separated into one part due to the sun and one part due to the sky. Denoting these by the superscripts "*sun*" and "*sky*" respectively, we can express (17) in more compact form as

$$
\begin{aligned}
N_h &= \left\{ N_h^{sun} + N_h^{sky} + N_h^{sea} \right\} \tau \, f + \left\{ N^{path}/2 \right\} f \\
N_v &= \left\{ N_v^{sun} + N_v^{sky} + N_v^{sea} \right\} \tau \, f + \left\{ N^{path}/2 \right\} f \\
N &= \left\{ N^{sun} + N^{sky} + N^{sea} \right\} \tau \, f + \left\{ N^{path} \right\} f
\end{aligned}
\tag{18}
$$

Thus, the received sea radiance consists of four independent components. Let us refer to each component by the superscript j where j is 1 for the sun, 2 for the sky, 3 for the sea, and 4 for the path. Total radiance will be referred to by omitting the superscript altogether. We define the degree of polarization of the jth component to be

$$
P^j \equiv \frac{N_h^j - N_v^j}{N_h^j + N_v^j} = \frac{N_h^j - N_v^j}{N^j} \tag{19}
$$

Then, given the total radiance and polarization of each component, N^j and P^j respectively, we can find all other quantities from

$$
\begin{aligned}
N_h^j &= N^j \left\{ \frac{1 + P^j}{2} \right\} \\
N_v^j &= N^j \left\{ \frac{1 - P^j}{2} \right\} \\
N &= \sum_j N^j \\
P &= \sum_j P^j \frac{N^j}{N}
\end{aligned}
\tag{20}
$$

From the last equation we see that the total polarization consists of the sum of the polarizations of each component weighted by the fractional radiance contained in that component.

DISCUSSION

If it were possible to observe a completely flat sea at a zenith angle near the Brewster angle, about 50 degrees, reflected light would be completely *horizontally* polarized because of the vanishing value of R_p in the plane of incidence, which is vertical in this case. On the other hand, the emitted black body radiation would be completely *vertically* polarized because A_p is a maximum at the Brewster angle (see equations (10) and figures 4 and 5). If reflection were to dominate emission, or vice versa, then a high degree of polarization would be observed. But if reflection and emission were comparable in intensity, as often happens in the long-wave infrared band, each would tend to cancel the other giving rise to rather weak polarization of the radiance from a perfectly flat sea.

Such an ideal condition is never approached in the real sea, however, even under the most calm of conditions. There is always a breeze, however slight, ruffling the surface and introducing some tilt to the capillary wave facets. An idea of how much tilt there is can be gathered from (16), which states that a wind speed of 1 m s^{-1} produces a distribution of wave tilts whose standard deviation is 3 degrees. Although this is small compared the width of the minimum in R_p at the Brewster angle, it is large compared to the angular width of the sun, about ¼ of a degree. Hence, although the lightly ruffled real sea under calm conditions would not display much of a reduction in its degree of polarization below that of a totally flat sea, the amount of sun glitter, if any, would be greatly reduced by virtue of a substantial reduction in the proportion of facets within the tolerance ellipse.

As the wind picks up, sky reflection and thermal emission will not change their strength substantially, but they will gradually become more weakly polarized because the profusion of facet tilts, involving a range of angles of incidence, will eliminate the sharp distinction between "*s*" and "*p*" components that occurs at the Brewster angle. The wind will also tilt planes of incidence as it tilts the facets, allowing both "*s*" and "*p*" components access to the axis of "easy passage" though the polarizer. With ever higher winds, the polarization of sky reflection and sea emission will gradually be washed away.

On the other hand, the polarization of the glint will *not* be washed away as the wind increases because, regardless of wind speed, the slopes inside the ellipse remain exactly the same. The sun is almost a point source: wave slopes very close to the specular reflection slope (those within the tolerance ellipse) will glint, others will not. From another point of view, one emphasized recently by Walker (1994), sun glints are an example of a very nonlinear slope-to-radiance transformation. Hence, contrary to the situation with sky reflection and sea emission, the polarization of sun glint will hardly change at all as the wind increases.

As far as the strength of sun glint is concerned, it can be expected to go through a *maximum* as the wind increases. The reason for this is as follows: In general, the tolerance ellipse will be located away from the origin of slope space where the interaction probability density is small. Hence, the height of the glint column (Zeisse, 1994a) will usually be small under calm conditions. As the wind picks up, the interaction probability density will broaden, increasing the height of the glint column. Finally, at high winds, the flattening of the interaction probability density, which accompanies its broadening, will decrease the height of the glint column.

Returning to the subject of polarization, path effects will reduce sea polarization for two reasons. First, the polarized radiance leaving the footprint will be attenuated by the path transmission. Second, unpolarized radiance will accumulate along the path with a corresponding dilution in the polarization of the total radiance.

In figure 6, the radiance of each component is shown as a function of look azimuth with respect to the azimuth of the sun. This figure is taken from a model[5] of the real sea. The spectral region is the long-wave band, 830 to 1250 cm^{-1} (8 to 12 μm) at low spectral resolution (10 cm^{-1}). Navy aerosols were used with a visibility of 10 km and a moderate coastal influence (an air mass parameter ICSTL of 5 in MODTRAN2). The solar zenith angle is 70 degrees. The view is directly into a wind of 10 ms^{-1} from a height of 20 m looking down onto the ocean surface at an angle of 1 degree with respect to the horizon. The range to the ocean footprint is 1150 m and the transmission along the path to the footprint is 82 percent. These are windy conditions with small path effects and strong sun glints. The sky polarization is +17.7 percent, the sea polarization is –9.2 percent, and the path is assumed to be unpolarized. The sun polarization varies slightly with azimuth, from +22.7 percent at an azimuth 8° away from the sun to +27.6 percent looking directly along the sun's azimuth.

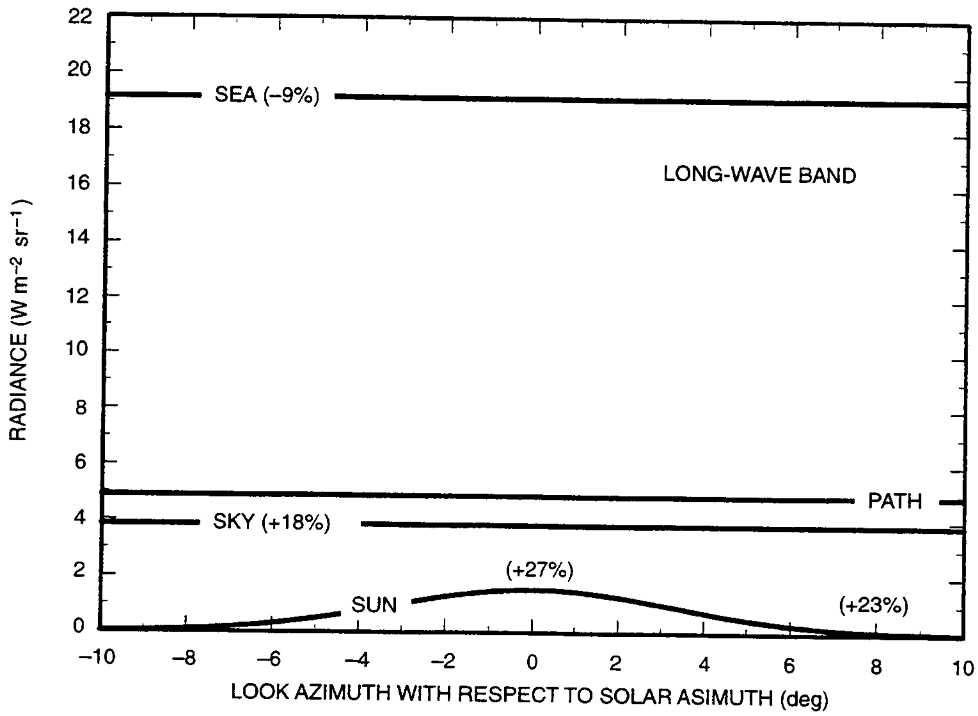

Figure 6. Sea radiance in the long-wave band for a rough sea. The wind speed is 10 m s^{-1}, the solar zenith angle is 70 degrees, and the view is directly up wind from a height of 20 m looking down at an angle of 1 degree with respect to the horizon. Numbers in parentheses refer to the polarization of each component.

Figure 7 shows the radiance in the mid-wave spectral band, 2000 to 3330 cm^{-1}, under the same sea conditions and viewing geometry. The transmission along the path to the footprint in this case is 53 percent. In this band, the path, sea, and sky radiance lie very close to one another; for clarity only their sum has been plotted in figure 7. The combined polarization of these three components is +5.1 percent using equation (19) with the following values of radiance and polarization for path, sea, and sky respectively: 0.797 W m^{-2} sr^{-1} and 0 percent, 0.423 W m^{-2} sr^{-1} and –13.9 percent, and

[5] Figures 6, 7, and 8 were produced by a polarized version of "SeaRad," a computer code for the prediction of sea radiance (Zeisse, 1995c).

0.529 W m^{-2} sr^{-1} and 28.2 percent. The polarization of the sun glint varies from +28.9 percent at an azimuth 8° away from the sun to +35.3 percent looking directly along the sun's azimuth.

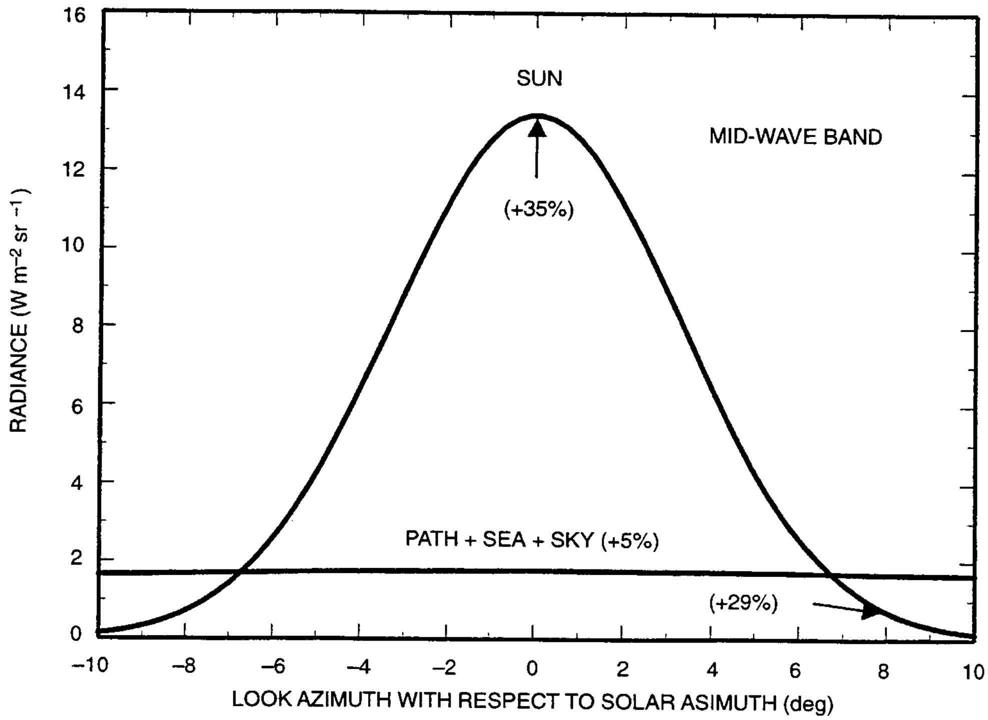

Figure 7. Sea radiance in the mid-wave band for a rough sea. The geometry is the same as for the previous figure. Numbers in parentheses refer to the polarization of each component.

Figure 8 shows the total polarization in each band for these conditions. In the mid-wave band, sun glint dominates all other components, and the total radiance along the center of the pattern is preferentially polarized by more than 30 percent in the horizontal direction. In the long-wave band, the solar spectrum is weak and the glints have diminished, leaving behind competing contributions from sky reflection, sea emission, and path emission. We see that in the long-wave band, the polarization of the total radiance is negligible for the case chosen here.

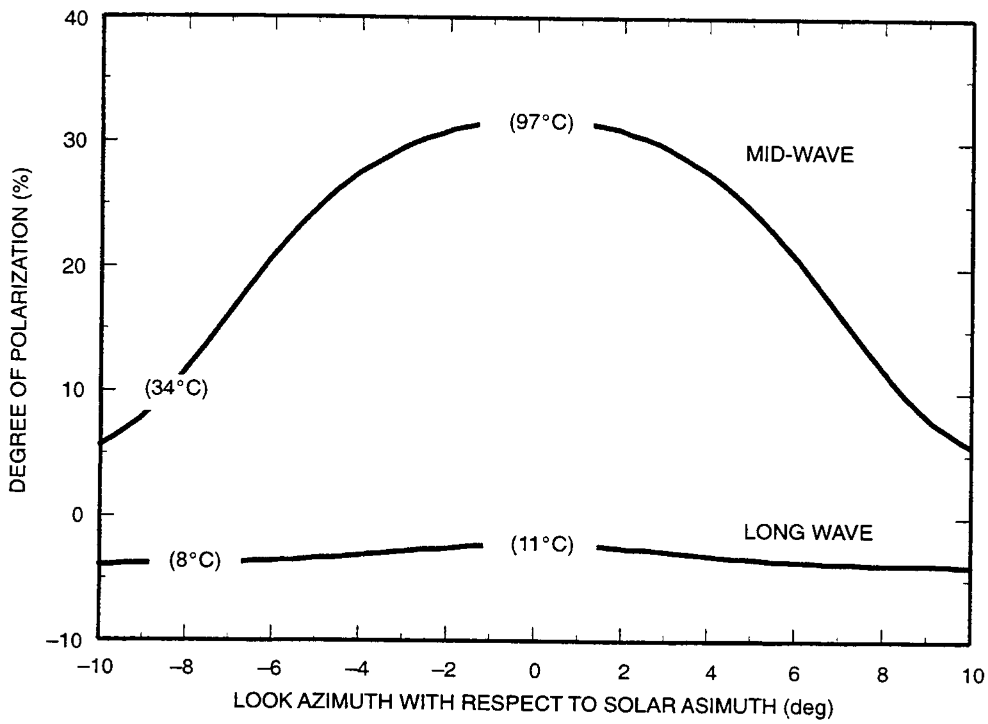

Figure 8. Sea polarization in the mid- and long-wave bands for a rough sea. The conditions are the same as for figure 6. A negative degree of polarization means that emission dominates reflection; a positive degree of polarization means that reflection dominates emission. Numbers in parentheses refer to the equivalent black body temperature of the corresponding signal.

CONCLUSION

For unpolarized incident radiance, horizontal and vertical sea radiance sum to unpolarized sea radiance.

It is remarkable that this relationship holds over the real sea since it reflects and emits from a bewildering variety of wave facet tilts. If "The sea surface is a mess" (Tropf, 1990), then "The polarized sea surface is a bigger mess," and it is amazing that an orderly relation such as this one has managed to survive.

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LIST OF SYMBOLS

LATIN

A	Unpolarized emission coefficient
$A_{s,p}$	Fresnel emission coefficients
$A_{h,v}$	Polarized emission coefficients
a	Unit vector component in the X direction
b	Unit vector component in the Y direction
c	Unit vector component in the Z direction
f	Relative spectral responsivity of receiver
$\mathbf{E}$	Amplitude of electromagnetic vector
$\mathbf{k}$	Unit vector in the Z direction.
n	Complex optical index.
N_o	Incident spectral radiance (W m^{-2} sr^{-1} (cm^{-1})$^{-1}$)
N_{bb}	Spectral black body radiance of the sea
$N_{h,v}$	Polarized total spectral radiance
N	Unpolarized total spectral radiance
$N_{s,p}$	Fresnel radiance
$N_{h,v}$	Polarized radiance
$R_{s,p}$	Fresnel reflection coefficients
$R_{h,v}$	Polarized reflection coefficients
R	Unpolarized reflection coefficient
$\mathbf{U}$	Unit vector
W	Wind speed (m s^{-1})
X	Coordinate axis in up wind direction
Y	Coordinate axis in cross wind direction
Z	Coordinate axis in zenith direction

GREEK

α	Angle of arrival
β	Angle between planes of incidence and vision
δ	Phase of plane wave
ϕ	Azimuth of unit vector
ν	Wave number
θ	Zenith angle of unit vector
ρ	Spectral amplitude reflectance ratio
σ	Slope variance
τ	Spectral path transmittance (footprint to receiver)
ω	Angle of incidence

SUBSCRIPTS

c	Cross wind
h	Horizontal
$imag$	Imaginary part of optical index
n	Facet normal
o	Incident (sky or sun)
p	Parallel to plane of incidence
r	Receiver
$real$	Real part of optical index
s	Perpendicular to plane of incidence
u	Up wind
v	Vertical
2	Reflected

SUPERSCRIPTS

path	Path radiance (footprint to receiver)
sc	Scattered solar sky radiance
sea	Thermal sea radiance
sky	Sky radiance reflected by the sea
sun	Solar radiance reflected by the sea (sun glint)
th	Thermal sky radiance

APPENDIX A: EQUATIONS FOR FRESNEL REFLECTION FROM A CONDUCTING SURFACE

Let electromagnetic waves of frequency ν propagate through a medium characterized by a magnetic susceptibility μ, a conductivity σ, and a dielectric constant ε. Then we have the following relationships (Stratton, 1941, pp. 268 ff):

$$\begin{aligned} k &= a + ib \\ a &= \frac{\varpi}{c} n_{real} = \frac{2\pi}{\lambda_o} n_{real} \\ b &= \frac{\varpi}{c} n_{imag} = \frac{2\pi}{\lambda_o} n_{imag} \\ a^2 - b^2 &= \mu\varepsilon\varpi^2 \\ ab &= \frac{\mu\sigma\varpi}{2} \end{aligned} \tag{A-1}$$

where k^2 is called the separation constant, ϖ is 2π times the frequency of the wave, λ_o is the free-space wavelength, and n_{real} and n_{imag} are the real and imaginary parts, respectively, of the optical index of the medium.

Consider the case in which there is Fresnel reflection at the boundary between two media, air (medium 2) and sea water (medium 1). Assume that air is a perfect dielectric ($\sigma_2 = 0$) and that sea water is a conductor ($\sigma_1 \neq 0$). Then we have these relationships (Stratton, 1941, page 501, equation (48)):

$$\begin{aligned} k_1^2 &= \varpi^2 \varepsilon_1 \mu_1 + i\varpi\sigma_1\mu_1 \\ k_1 &= a_1 + ib_1 \end{aligned} \tag{A-2}$$

for the water, where a_1 and b_1 are given in terms of the material constants or optical indices of water by (A-1), and these relationships:

$$\begin{aligned} k_2^2 &= \varpi^2 \varepsilon_2 \mu_2 \\ k_2 &= a_2 = \varpi\sqrt{\varepsilon_2\mu_2} \end{aligned} \tag{A-3}$$

for the air.

Let ω be the angle of incidence and define p and q (Stratton, 1941, page 505, equation (69)) by

$$\begin{aligned} q^2 - p^2 &\equiv a_1^2 - b_1^2 - a_2^2 \sin^2\omega \\ q^2 + p^2 &\equiv \sqrt{(2a_1b_1)^2 + (a_1^2 - b_1^2 - a_2^2 \sin^2\omega)^2} \end{aligned} \tag{A-4}$$

Then (Stratton, 1941, page 505, equation (74))

$$\rho_s^2 = \frac{[\mu_2 q - \mu_1 a_2 \cos\omega]^2 + \mu_2^2 p^2}{[\mu_2 q + \mu_1 a_2 \cos\omega]^2 + \mu_2^2 p^2} \equiv R_s$$

$$\tan\delta_s = \frac{2\mu_1\mu_2 a_2 p \cos\omega}{\mu_1^2 a_2^2 \cos^2\omega - \mu_2^2 (q^2 + p^2)} \qquad \text{(A-5)}$$

and (Stratton, 1941, page 506, equation (77))

$$\rho_p^2 = \frac{\left[\mu_2 (a_1^2 - b_1^2)\cos\omega - \mu_1 a_2 q\right]^2 + [2\mu_2 a_1 b_1 \cos\omega - \mu_1 a_2 p]^2}{\left[\mu_2 (a_1^2 - b_1^2)\cos\omega + \mu_1 a_2 q\right]^2 + [2\mu_2 a_1 b_1 \cos\omega + \mu_1 a_2 p]^2} \equiv R_p$$

$$\tan\delta_p = \frac{2\mu_1\mu_2 a_2 p (q^2 + p^2 - a_2^2 \sin^2\omega)\cos\omega}{\mu_1^2 a_2^2 (q^2 + p^2) - \mu_2^2 (a_1^2 + b_1^2)\cos^2\omega} \qquad \text{(A-6)}$$

and when $\mu_1 = \mu_2$ (Stratton, 1941, page 506, equation (80)).

$$\rho^2 \equiv \frac{\rho_p^2}{\rho_s^2} = \frac{[q - a_2 \sin\omega \tan\omega]^2 + p^2}{[q + a_2 \sin\omega \tan\omega]^2 + p^2}$$

$$\tan\delta \equiv \tan(\delta_p - \delta_s) = \frac{2a_2 p \sin\omega \tan\omega}{a_2^2 \sin^2\omega \tan^2\omega - (q^2 + p^2)} \qquad \text{(A-7)}$$

When a and b are expressed in terms of the real and imaginary parts of the optical index of the medium using (A-1), the factor $2\pi/\lambda_o$ cancels from each of equations (A-5) through (A-7). Hence in these equations, $2\pi/\lambda_o$ can be set equal to one, and the real part of the index can be used in place of a, and the imaginary part of the index can be used in place of b. With these substitutions, and with

$$\begin{aligned} \mu_1 &= \mu_2 \\ n_{2real} &\approx 1 \\ n_{1real} &\equiv n_{real} \\ n_{1imag} &\equiv n_{imag} \end{aligned} \qquad \text{(A-8)}$$

the equations for q and p become

$$q^2 - p^2 = n_{real}^2 - n_{imag}^2 - \sin^2\omega$$

$$q^2 + p^2 = \sqrt{(2n_{real}n_{imag})^2 + (n_{real}^2 - n_{imag}^2 - \sin^2\omega)^2} \qquad \text{(A-9)}$$

the perpendicular reflection coefficient and phase become

$$\rho_s^2 = \frac{[q - \cos\omega]^2 + p^2}{[q + \cos\omega]^2 + p^2} \equiv R_s$$

$$\tan\delta_s = \frac{2p\cos\omega}{\cos^2\omega - (q^2 + p^2)} \qquad \text{(A-10)}$$

the parallel reflection coefficient and phase become

$$\rho_p^2 = \frac{\left[\left(n_{real}^2 - n_{imag}^2\right)\cos\omega - q\right]^2 + \left[2n_{real}n_{imag}\cos\omega - p\right]^2}{\left[\left(n_{real}^2 - n_{imag}^2\right)\cos\omega + q\right]^2 + \left[2n_{real}n_{imag}\cos\omega + p\right]^2} \equiv R_p$$

$$\tan\delta_p = \frac{2p\left(q^2 + p^2 - \sin^2\omega\right)\cos\omega}{q^2 + p^2 - \left(n_{real}^2 + n_{imag}^2\right)\cos^2\omega} \qquad \text{(A-11)}$$

and

$$\rho^2 \equiv \frac{\rho_p^2}{\rho_s^2} = \frac{\left[q - \sin\omega\ \tan\omega\right]^2 + p^2}{\left[q + \sin\omega\ \tan\omega\right]^2 + p^2}$$

$$\tan\delta \equiv \tan\left(\delta_p - \delta_s\right) = \frac{2p\sin\omega\tan\omega}{\sin^2\omega\ \tan^2\omega - \left(q^2 + p^2\right)} \qquad \text{(A–12)}$$

"When one considers that the quantities q and p are functions of the angle of incidence as well as of all the parameters of the medium, the complexity of what appeared at first to be the simplest of problems—the reflection of a plane wave from a plane, absorbing surface—is truly amazing. The formulas are far too involved to be understood from a casual inspection, and the nature of the reflection phenomena becomes apparent only when one treats certain limiting cases." (Stratton, 1941, page 507).

REFERENCE

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APPENDIX B: FORTRAN CALCULATION OF FRESNEL REFLECTION FROM SEA WATER

```
0       SUBROUTINE Rho(omega, v, Rs, Rp)
CU      USES SeaData
        COMMON /SeaIndex/ alpha01(100), alpha02(20),
     +                    beta01 (100), beta02 (20)
**********************************************************************
*                                                                    *
*       Calculates reflectivity of sea water between 52.63 cm-1 and  *
*  25,000 cm-1 using equations (74) and (78) of Stratton, "Electro-  *
*  magnetic Theory," 1941, page 505, ff.  The sea water is assumed to *
*  be a conducting medium;  both real and imaginary parts of the     *
*  index of water are used.  The notation of Stratton is adhered to  *
*  as closely as possible except that I have set alpha equal to the  *
*  real part of the index [rather than (2*pi)/lambda*Real(n)] and beta *
*  equal to the imaginary part of the index, [rather than            *
*  (2*pi)/lambda*Imag(n)], which can be done because the factor      *
*  (2*pi)/lambda cancels out in these equations. Lambda is the free  *
*  space wavelength.                                                 *
*                                                                    *
*  The calculation is for a single flat wave facet, sea water on one *
*  side and air on the other side.                                   *

*                                                                    *
*       Omega is the angle of incidence in radians, and v is the wave *
*  number in cm-1; these are both inputs. The outputs are Rp and Rs. *
*  Rp is the reflectivity that would be observed with a polarizer    *
*  oriented parallel to the plane of incidence. Rs is the reflectivity *
*  that would be observed with a polarizer oriented perpendicular to *
*  the plane of incidence. To find the unpolarized reflectivity, take *
*  (Rp + Rs)/2.                                                      *
*                                                                    *
*  Last revision:  December 28, 1995                                 *
*                                                                    *
```

```
*************************************************************************
C     The four-point interpolation functions are:
      WM(x) = (x - 1.)*(x - 2.)*x/6.
      W0(x) = (x - 1.)*(x - 2.)*(x + 1.)/2.
      W1(x) = (x + 1.)*(x - 2.)*x/2.
      W2(x) = (x + 1.)*(x - 1.)*x/6.
      If ((omega .lt. 0.) .or. (omega .gt. 1.57080)) then
C     omega is out of bounds; set reflectivities to 0% and return:
          Rp = 0.
          Rs = 0.
          Return
      End If
      If (v .eq. 0.) then
C     set reflectivities to 100% and return:
          Rp = 1.
          Rs = 1.
          Return
      End If
      W = 1.E4/v
      If (W .LT. 0.399999) then
C     print error message and return:
          Rp = 0.
          Rs = 0.
          Write (6, 1000) v
          Return
      End If
      If (0.4 .le. W .and. W .le. 19.8) then
C     use Lagrange 4 point interpolation on Block 01 data which
C     are at 0.2 um spacing between 0.2 and 20 um:
          I  = Int(W/0.2)
          Fr = Mod(W, 0.2)/0.2
          alpha1 = W2(Fr)*alpha01(I + 2) - W1(Fr)*alpha01(I + 1)
    +            + W0(Fr)*alpha01(I)     - WM(Fr)*alpha01(I - 1)
          beta1  = W2(Fr)*beta01 (I + 2) - W1(Fr)*beta01 (I + 1)
```

```
+             + W0(Fr)*beta01 (I)      - WM(Fr)*beta01 (I - 1)
      End If
      If (19.8 .lt. W .and. W .lt. 190.) then
C     use Lagrange 4 point interpolation on Block 02 data which
C     are at 10 um spacing between 10 and 200 um:
          I  = Int(W/10.)
          Fr = Mod(W, 10.)/10.
          alpha1 = W2(Fr)*alpha02(I + 2) - W1(Fr)*alpha02(I + 1)
+              + W0(Fr)*alpha02(I)      - WM(Fr)*alpha02(I - 1)
          beta1  = W2(Fr)*beta02  (I + 2) - W1(Fr)*beta02  (I + 1)
+              + W0(Fr)*beta02  (I)      - WM(Fr)*beta02  (I - 1)
      End If
      If (190. .LE. W) then
C     print error message and return:
          Rp = 0.
          Rs = 0.
          Write (6, 1000) v
          Return
      End If
      g = alpha1**2 - beta1**2 - sin(omega)**2
      h = 4.*(alpha1**2)*(beta1**2)
      p = sqrt(0.5*(-g + sqrt(h + g**2)))
      q = sqrt(0.5*(+g + sqrt(h + g**2)))
C     Stratton, Equation (74), p. 505:
      c  = (q - cos(omega))**2 + p**2
      d  = (q + cos(omega))**2 + p**2
      Rs = c/d
C     Stratton, Equation (77), p. 506:
      e  = ((alpha1**2 - beta1**2)*cos(omega) - q)**2
      f  = ((alpha1**2 - beta1**2)*cos(omega) + q)**2
      t  = (2*alpha1*beta1*cos(omega) - p)**2
      u  = (2*alpha1*beta1*cos(omega) + p)**2
      Rp = (e + t)/(f + u)
      Return
```

```
 1000 FORMAT (' ***** WARNING - INPUT FREQUENCY = ', 1PG12.5, 'CM-1',
     +        /, ' OUTSIDE VALID RANGE OF 52.63 TO 25,000 CM-1  ******',/)
      End
      BLOCK DATA SeaData
      COMMON /SeaIndex/ alpha01(100), alpha02(20),
     +                  beta01 (100), beta02 (20)
************************************************************************
*                                                                      *
*   These data for the optical index of water have been taken from     *
*   the literature.  From 0.2 to 2 microns (blocks 01 up to second     *
*   entry of row B) and from 10 to 200 microns (blocks 02) the data    *
*   are from G. M. Hale and M. R. Querry, "Optical Constants of Water  *
*   in the 200-nm to 200-um Wavelength Region," Appl. Opt. 3, 555      *
*   (1973).  These data are for pure water.                            *
*                                                                      *
*   From 2.2 to 20 microns (blocks 01 from the third entry of row B    *
*   to the end), the data are from M. R. Querry, W. E. Holland, R. C.  *
*   Waring, L. M. Earls, and M. D. Querry, "Relative Reflectance and   *
*   Complex Refractive Index in the Infrared for Saline Environmental  *
*   Waters," J. Geophys. Res. 82, p. 1425 (1977), Table 3, Pacific     *
*   Ocean columns.  These data are for salt water.                     *
*                                                                      *
************************************************************************
C     Real part of the index of sea water from 0.2 to 20 microns in
C     steps of 0.2 microns:
      DATA alpha01  /
     A     1.396, 1.339, 1.332, 1.329, 1.327, 1.324, 1.321, 1.317,
     B     1.312, 1.306, 1.303, 1.287, 1.251, 1.151, 1.384, 1.479,
     C     1.421, 1.388, 1.368, 1.355, 1.347, 1.339, 1.335, 1.335,
     D     1.332, 1.324, 1.312, 1.296, 1.268, 1.271, 1.371, 1.353,
     E     1.340, 1.330, 1.324, 1.319, 1.314, 1.307, 1.302, 1.297,
     F     1.291, 1.286, 1.279, 1.272, 1.268, 1.264, 1.258, 1.249,
     G     1.240, 1.229, 1.218, 1.204, 1.190, 1.177, 1.165, 1.151,
     H     1.140, 1.132, 1.124, 1.119, 1.121, 1.126, 1.134, 1.142,
```

```
     I        1.152, 1.164, 1.177, 1.189, 1.201, 1.212, 1.224, 1.234,
     J        1.242, 1.253, 1.261, 1.273, 1.284, 1.296, 1.309, 1.320,
     K        1.331, 1.339, 1.349, 1.358, 1.366, 1.379, 1.390, 1.399,
     L        1.408, 1.417, 1.426, 1.435, 1.443, 1.450, 1.458, 1.464,
     M        1.470, 1.474, 1.477, 1.480  /
C     Real part of the index of sea water from 10 to 200 microns in
C     steps of 10 microns:
      DATA alpha02  /
     A        1.218, 1.480, 1.551, 1.519, 1.587, 1.703, 1.821, 1.886,
     B        1.924, 1.957, 1.966, 2.004, 2.036, 2.056, 2.069, 2.081,
     C        2.094, 2.107, 2.119, 2.130  /
C     Imaginary part of the index of sea water from 0.2 to 20 microns in
C     steps of 0.2 microns:
      DATA beta01  /
     A        0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000, 0.000,
     B        0.000, 0.001, 0.000, 0.001, 0.003, 0.114, 0.263, 0.085,
     C        0.018, 0.005, 0.003, 0.004, 0.007, 0.011, 0.016, 0.016,
     D        0.013, 0.011, 0.010, 0.013, 0.032, 0.108, 0.087, 0.044,
     E        0.035, 0.033, 0.032, 0.031, 0.031, 0.032, 0.032, 0.033,
     F        0.034, 0.036, 0.038, 0.041, 0.044, 0.046, 0.046, 0.047,
     G        0.048, 0.050, 0.054, 0.060, 0.068, 0.079, 0.091, 0.107,
     H        0.125, 0.145, 0.166, 0.191, 0.216, 0.239, 0.260, 0.279,
     I        0.297, 0.313, 0.326, 0.338, 0.347, 0.357, 0.363, 0.371,
     J        0.377, 0.385, 0.393, 0.401, 0.407, 0.413, 0.417, 0.418,
     K        0.420, 0.422, 0.424, 0.427, 0.430, 0.432, 0.432, 0.432,
     L        0.431, 0.430, 0.429, 0.427, 0.425, 0.423, 0.420, 0.416,
     M        0.414, 0.410, 0.406, 0.393  /
C     Imaginary part of the index of sea water from 10 to 200 microns in
C     steps of 10 microns:
      DATA beta02  /
     A        0.051, 0.393, 0.328, 0.385, 0.514, 0.587, 0.576, 0.547,
     B        0.536, 0.532, 0.531, 0.526, 0.514, 0.500, 0.495, 0.496,
     C        0.497, 0.499, 0.501, 0.504  /
      End
```

APPENDIX C: THE ANGLE BETWEEN THE PLANES OF INCIDENCE AND VISION

In this appendix we derive an expression for β, the angle between the plane of incidence and the plane of vision. Let $\mathbf{U}_p$ be a unit vector normal to $\mathbf{U}_r$ pointing along the direction of $\mathbf{E}_{2p}$ in the plane of incidence and let $\mathbf{U}_v$ be a unit vector normal to $\mathbf{U}_r$ pointing along the vertical axis of polarization in the plane of vision. Then (see figure 3)

$$\cos\beta = \mathbf{U}_p \cdot \mathbf{U}_v \tag{C-1}$$

To find β we must find the coordinates of $\mathbf{U}_p$ and $\mathbf{U}_v$ and then take their inner product.

First we will find the coordinates of $\mathbf{U}_p$. Since it is normal to $\mathbf{U}_r$

$$\mathbf{U}_p \cdot \mathbf{U}_r = 0 \tag{C-2}$$

and since it is in the plane of incidence, it can be written as a linear combination of two other vectors in that plane:

$$\mathbf{U}_p = A\mathbf{U}_n + B\mathbf{U}_r \tag{C-3}$$

Setting the inner product of (C-3) with $\mathbf{U}_r$ equal to zero, recalling that ω is the angle between $\mathbf{U}_n$ and $\mathbf{U}_r$, and normalizing the length of $\mathbf{U}_p$ to one, we find that

$$\pm\mathbf{U}_p = \frac{\mathbf{U}_n - \cos\omega\ \mathbf{U}_r}{\sin\omega} \tag{C-4}$$

Next we want the coordinates of $\mathbf{U}_v$. These might appear to be trivially available since $\mathbf{U}_v$ is "vertical" and coincident with the unit vector **k** along the Z axis with coordinates (0, 0, 1). However, $\mathbf{U}_v$ is *not* vertical since it is normal to $\mathbf{U}_r$, a vector that is generally not horizontal. The coordinates of $\mathbf{U}_v$ must be found by considering how the polarizer is actually adjusted in practice: first, the polarizer is set horizontal (no Z component) and, second, the polarizer is turned by 90 degrees. So, to find the coordinates of $\mathbf{U}_v$, we have to find the coordinates of $\mathbf{U}_h$ and then construct $\mathbf{U}_v$ normal to $\mathbf{U}_h$ and $\mathbf{U}_r$. Since $\mathbf{U}_h$ is normal to $\mathbf{U}_r$

$$\mathbf{U}_h \cdot \mathbf{U}_r = 0 \tag{C-5}$$

and since $\mathbf{U}_h$ is horizontal

$$\mathbf{U}_h \cdot \mathbf{k} = 0. \tag{C-6}$$

These conditions, plus the requirement that the length of $\mathbf{U}_h$ be one result, after some algebra, in the coordinates

$$\pm\mathbf{U}_h = (\sin\phi_r,\ -\cos\phi_r,\ 0) \tag{C-7}$$

The calculation for $\mathbf{U}_v$ follows similar lines but will be avoided altogether by noting that, as shown in figure 3, β is the complement of the angle between $\mathbf{U}_p$ and $\mathbf{U}_h$:

$$\sin\beta = \cos(\pi/2 - \beta) = \mathbf{U}_p \cdot \mathbf{U}_h = \pm\frac{\mathbf{U}_n \cdot \mathbf{U}_h}{\sin\,\omega} \tag{C-8}$$

where we have used the fact that $\mathbf{U}_h$ is normal to $\mathbf{U}_r$. According to (1) the coordinates of $\mathbf{U}_n$ are

$$\mathbf{U}_n = (\sin\theta_n \cos\phi_n,\ \sin\theta_n \sin\phi_n,\ \cos\theta_n) \tag{C-9}$$

and substitution of (C-9) and (C-7) into (C-8) gives

$$\sin\beta = \frac{\sin\theta_n \sin(\phi_r - \phi_n)}{\sin\omega}. \tag{C-10}$$

for the angular separation between the planes of incidence and vision.

APPENDIX D: POLARIZATION OF THE INFRARED SKY

The purpose of this appendix is to estimate the wave numbers at which the total polarization of the infrared sky reaches 1 percent and 10 percent. The reader is referred to (McCartney, 1976) for a more complete discussion of scattered solar radiance.

In the infrared the primary source of sky radiance is thermal emission of the atmosphere. In the visible the primary source of sky radiance is scattered sunlight. We will assume that for all wave numbers ν throughout the infrared and visible, the total spectral sky radiance $N_o(\nu)$ consists of a combination of spectral thermal radiance $N^{th}{}_o(\nu)$ and spectral scattered solar radiance $N^{sc}{}_o(\nu)$. Then equation (20) gives for the total sky polarization

$$P_o(\nu) = \frac{P_o^{sc}(\nu)\, N_o^{sc}(\nu) + P_o^{th}(\nu)\, N_o^{th}(\nu)}{N_o^{sc}(\nu) + N_o^{th}(\nu)} \tag{D-1}$$

where $P^{sc}{}_o(\nu)$ is the polarization of the scattered solar radiance and $P^{th}{}_o(\nu)$ is the polarization of the thermal radiance. The polarization of the scattered solar radiance varies between −10 percent and +90 percent depending on the observation direction (McCartney, 1976, page 212), and for the purposes of this estimate we shall assume that it is always +100 percent. The polarization of the thermal radiance vanishes since thermal radiance is a natural source. Hence the problem posed above becomes the problem of finding the values of ν for which

$$P_o(\nu) \approx \frac{N_o^{sc}(\nu)}{N_o^{sc}(\nu) + N_o^{th}(\nu)} \tag{D-2}$$

is equal to 1 percent and 10 percent.

The thermal spectral radiance can be approximated by the spectral radiance of a black body at absolute temperature 300 K:

$$N_o^{th} \approx N_{bb}(300K, \nu) \tag{D-3}$$

The following expression has been given (McCartney, 1976, p. 208) for single scattered solar radiance found at the bottom of a plane parallel atmosphere when the sunlight arriving at the top of the atmosphere has a spectral irradiance $E_o(\nu)$:

$$N_o^{sc}(\nu, \theta) = \frac{E_o(\nu)\beta_m(\theta)}{\beta_m}\left[\frac{\exp(-\beta_m Z_o' \sec\zeta_s) - \exp(-\beta_m Z_o' \sec\zeta_p)}{1 - \sec\zeta_s \cos\zeta_p}\right] \tag{D-4}$$

In this expression, θ is the supplement of the angle between the direction to the sun and the direction to the point of observation in the sky; ζ_s is the zenith angle of the solar direction; ζ_p is the zenith angle of the observation direction; and Z_o' is the reduced height of the atmosphere. The volume angular scattering coefficient, $\beta_m(\theta)$, is the intensity scattered into an infinitesimal solid angle $d\omega$ at an observation angle θ with respect to the direction of the incident light. For unpolarized incident light, it is given by

$$\beta_m(\theta) = \frac{\pi^2 (n^2 - 1)^2 \nu^4}{2N} (1 + \cos^2\theta) \tag{D-5}$$

where n is the refractive index of the molecular atmosphere consisting of N dipoles per unit volume. Finally, the total volume scattering coefficient

$$\beta_m = \int_0^{4\pi} \beta_m(\theta)\, d\omega = \frac{8\pi^3 (n^2 - 1)^2 \nu^4}{3N} \tag{D-6}$$

is the ratio of the flux scattered in all directions, by unit volume of gas, to the irradiance of the incident flux[1]. The use of a plane parallel atmosphere causes errors of less than 2 percent for all zenith angles ζ less than 80 degrees (sec ζ less than 6).

We wish to show that the argument of each exponential in (D-4) is small. Values of $(n^2 - 1)^2$ are weakly dependent on wave number and stay within several percent of 3 x 10^{-7} throughout the infrared and visible (McCartney 1976, page 348). N is given by the number of molecules per unit volume in the uniform reduced atmosphere at standard temperature and pressure, namely, 2.547 x 10^{19} cm^{-3}. This means that the value of β_m is approximately 10^{-3} km^{-1} at a wave number of 10,000 cm^{-1}. Next, the reduced height of the 1962 Standard Atmosphere is 8.00 km (McCartney, 1976, page 92), and sec ζ is less than 6, so the arguments of the two exponentials in (D-4) are indeed less than one in the infrared and we can approximate (D-4) by

$$N_o^{sc}(\nu,\theta) \approx E_o^{sun} \beta_m(\theta)\, Z_o' \left[\frac{\sec\zeta_p - \sec\zeta_s}{1 - \sec\zeta_s \cos\zeta_p} \right] \tag{D-7}$$

Furthermore, we can replace $(1 + \cos^2\theta)$ in (D-5) by its maximum value of 2, neglect the square bracket in (D-7) whose value is on the order of unity, and express the solar irradiance at the top of the atmosphere in terms of the solar radiance there times the solid angle ω_o subtended by the sun, which is 6.763 x 10^{-5} sr. We thus arrive at

$$N_o^{sc}(\nu) \approx N_{bb}(5900\,K,\nu)\cdot\omega_o\cdot\frac{\pi^2 (n^2 - 1)^2 \nu^4}{N}\cdot Z_o' \approx 6.29\cdot 10^{-24}\ cm^4 \cdot N_{bb}(5900\,K,\nu)\cdot\nu^4 \tag{D-8}$$

for the spectral radiance of a plane parallel Rayleigh sky at the surface of the earth in the infrared visible spectral regions. In (D-8) we have gone on to assume that the solar spectral radiance is given by Planck's equation for a black body $N_{bb}(T, \nu)$ at absolute temperature T = 5900 K.

[1] In expressions (D-5) and (D-6), it is helpful to remember that $n^2 - 1$ is closely proportional to N so that the scattering coefficients are proportional to the number of scattering centers N per unit volume as expected.

Table D-1 lists some values of spectral sky radiance according to (D-8) and its polarization according to (D-2):

Table D-1. Spectral sky radiance and polarization.

ν	$N_{bb}(5900\ K, \nu)$	$N_o^{sc}(\nu)$	$N_{bb}(300\ K, \nu)$	$P(\nu)$
(cm^{-1})	($W\ m^{-2}\ sr^{-1}\ (cm^{-1})^{-1}$)	($W\ m^{-2}\ sr^{-1}\ (cm^{-1})^{-1}$)	($W\ m^{-2}\ sr^{-1}\ (cm^{-1})^{-1}$)	(%)
3,000	269	1.37×10^{-7}	1.64×10^{-4}	0.1
3,500	341	3.22×10^{-7}	2.37×10^{-5}	1.3
4,000	415	6.68×10^{-7}	3.23×10^{-6}	17.1
4,500	489	1.26×10^{-6}	4.18×10^{-7}	75.1
5,000	562	2.21×10^{-6}	5.22×10^{-8}	97.7

Hence the required estimate is

$$
\begin{aligned}
P(\nu) &\approx 1\%\ at\ \nu \approx 3{,}500\ cm^{-1}\ (2.9\ \mu m) \\
P(\nu) &\approx 10\%\ at\ \nu \approx 4{,}000\ cm^{-1}\ (2.5\ \mu m)
\end{aligned}
\qquad \text{(D-9)}
$$

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